

- Q_s : extends on the semiconductor
- insulator: no charges (ideal)
- metal: $\vec{E} = 0 \Rightarrow$ charges only at surface

Gauss Law:

$$\frac{\partial E}{\partial x} = \frac{\rho(x)}{\epsilon} \Rightarrow E = \frac{Q}{\epsilon} \quad \begin{array}{l} \text{total} \\ \text{charge} \\ \text{density} \end{array} \quad \text{dielectric constant}$$

- Electric field is discontinuous but $\vec{D}$ normal must be continuous:
 $\epsilon_s \cdot E_s = \epsilon_{ox} \cdot E_{ox}$ at interface

Capacitance per unit area:

$$C_{ox} = \frac{\epsilon_{ox}}{\lambda_{ox}} \quad \begin{array}{l} \text{dielectric constant} \\ \text{thickness of oxide} \end{array}$$

* Other ways of looking at this: #1

$$E = F/q =$$

$$C[F] = \frac{Q[C]}{V}$$

thus $E \cdot \left(\frac{V}{cm} \right) = \frac{Q}{cm^2} \Rightarrow |\vec{E}| = \frac{Q_s}{\epsilon}$ charge density

or $C = \frac{\epsilon \cdot A}{t}$

$$Q = C \cdot V$$

thus:

$$\frac{Q}{A} = \frac{\epsilon \cdot V}{t \cdot |\vec{E}|}$$

$$\Rightarrow Q_s = \epsilon \cdot |\vec{E}|$$

$\therefore$ the dielectric constant links the amount of charges to an electric field!

Exercise 1:

Consider MOS structure:

$$\lambda_{ox} = 4.5 \text{ nm}$$

$$\phi_{bi} = 1 \text{ V}$$

$$\phi_{ox} = 0.42 \text{ V at zero bias}$$

calculate ϕ_s , Q_s , E_s , E_{ox} :

$$E_{ox} = \frac{\phi_{ox}}{\lambda_{ox}} = \frac{0.42 \text{ V}}{4.5 \times 10^{-7} \text{ cm}} = 9.3 \times 10^5 \text{ V/cm}$$

$$E_s = \frac{\epsilon_{ox} \cdot E_{ox}}{\epsilon_s} = \frac{3.9}{11.7} \times 9.3 \times 10^5 = 3.1 \times 10^5 \text{ V/cm}$$

$$Q_s = \epsilon_s E_s = 11.7 \times 8.85 \times 10^{-14} \text{ F/cm} \times 3.1 \times 10^5 \text{ V/cm} = 3.2 \times 10^{-7} \text{ C/cm}^2$$

$$\phi_s = \phi_{bi} - \phi_{ox} = 0.58 \text{ V}$$

$$C_{ox} = \frac{\epsilon_{ox}}{x_{ox}} = \frac{3.9 \times 8.85 \times 10^{-14} \text{ F/cm}}{4.5 \times 10^{-7} \text{ cm}} = 7.7 \times 10^{-7} \text{ F/cm}^2$$

$$\gamma = \frac{1}{C_{ox}} \sqrt{2 \epsilon_s q N_A} = \frac{\sqrt{2 \times 1.04 \times 10^{-12} \text{ F/cm} \times 1.6 \times 10^{-19} \text{ C} \times 6 \times 10^{17} \text{ cm}^{-3}}}{7.7 \times 10^{-7} \text{ F/cm}^2}$$

$$\gamma = 0.58 \text{ V}^{1/2}$$

$$\epsilon_s = 11.7 \times 8.85 \times 10^{-14} \text{ F/cm}$$

thus:

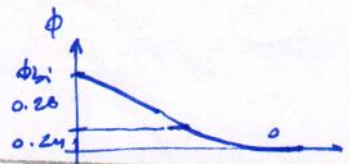
$$x_d = \frac{\epsilon_s}{C_{ox}} \left(\sqrt{1 + \frac{4 \phi_{bi}}{\gamma^2}} - 1 \right) = \frac{1.04 \times 10^{-12} \text{ F/cm}}{7.7 \times 10^{-7} \text{ F/cm}^2} \cdot \left(\sqrt{1 + \frac{4 \times 0.52}{0.58^2}} - 1 \right) = 23 \text{ nm}$$

$$E_{ox} = \frac{q N_A x_d}{\epsilon_{ox}} = \frac{1.6 \times 10^{-19} \text{ C} \cdot 6 \times 10^{17} \text{ cm}^{-3} \times 2.3 \times 10^{-6} \text{ cm}}{3.45 \times 10^{-13} \text{ F/cm}} = 6.4 \times 10^5 \text{ V/cm}$$

$$\Rightarrow \text{thus } \phi_{ox} = E_{ox} \cdot x_{ox} = 6.4 \times 10^5 \text{ V/cm} \times 4.5 \times 10^{-7} \text{ cm} = 0.288 \text{ V}$$

$$\phi_s = \frac{q N_A x_d^2}{2 \epsilon_s} = \frac{1.6 \times 10^{-19} \text{ C} \cdot 6 \times 10^{17} \text{ cm}^{-3} \times (2.3 \times 10^{-6})^2}{2 \times 1.04 \times 10^{-12} \text{ F/cm}} = 0.24 \text{ V}$$

$$\Rightarrow \phi_{bi} = 0.24 \text{ V}$$



MOS under bias:

$$\phi_{bi} \longrightarrow \phi_{bi} + V$$

presence of the oxide blocks the current $\Rightarrow$ electrostatics are similar to that of a pn diode or Schottky diode

$$\Rightarrow \phi_{bi} + V = \phi_s(V) - \frac{Q_s(V)}{C_{ox}}$$

Exercise: Consider on MOS structure with:

#4

- Poly-Si gate $\chi_M = 4.04 \text{ eV}$
- SiO_2 oxide : 4.5 nm
- p-type $N_A = 6 \times 10^{17} \text{ cm}^{-3}$

at $V = -0.5 \text{ V}$.

Compute Q_s and χ_d ($V = -0.5 \text{ V}$) & V_{th} , Q_i (for $V = 2 \text{ V}$). How about the carrier density?

1) previously $\phi_{bi} = 1 \text{ V}$, $C_{ox} = 7.7 \times 10^{-7} \text{ F/cm}^2$

2) previously : $\gamma = 0.58 \text{ V}$

Depletion charge : $Q_s = -\frac{1}{2} \gamma^2 C_{ox} \left(\sqrt{1 + 4 \frac{\phi_{bi} + V}{\gamma^2}} - 1 \right)$

$$= -\frac{1}{2} \cdot (0.58)^2 \cdot 7.7 \times 10^{-7} \text{ F/cm}^2 \cdot \left[\sqrt{1 + 4 \frac{1.0 - 0.5}{(0.58)^2}} - 1 \right] = -2.1 \times 10^{-7} \text{ C/cm}^2$$

$$\chi_d = \frac{Q_s}{-q \cdot N_A} = \frac{-2.1 \times 10^{-7}}{-1.6 \times 10^{-19} \times 6 \times 10^{17}} = 22 \text{ nm}$$

3) $\phi_{sth} = \frac{2 kT}{q} \ln \frac{N_A}{n_i} = 2 \times 0.026 \times \ln \frac{6 \times 10^{17}}{1.02 \times 10^{10}} = 0.93 \text{ V}$

$$V_{th} = \phi_{FB} + \phi_{sth} + \gamma \sqrt{\phi_{sth}}$$

$$= -1 \text{ V} + 0.93 + 0.58 \cdot \sqrt{0.93} = 0.49 \text{ V}$$

$$\checkmark > 0.49 \text{ V}$$

$\Rightarrow$ inversion

for $V = 2 \text{ V}$:

$$Q_i = -C_{ox} \cdot (V - V_{th}) = -7.7 \times 10^{-7} \text{ F/cm}^2 \times (2 - 0.49) = -1.2 \times 10^{-6} \text{ C/cm}^2$$

$$\therefore n_i = \frac{-1.2 \times 10^{-6} \text{ C/cm}^2}{-1.6 \times 10^{-19}} = 7.5 \times 10^{12} \text{ cm}^{-2}$$